José Enrique Rodríguez Hernández

Tema 3. Potencias y raíces

Ejercicios resueltos

1. Simplificar:

a) $\left(-\frac{1}{2}\right)^4 \cdot \left(\frac{2}{9}\right)^{-1} \cdot \frac{1}{8} = \frac{1}{2^4} \cdot \frac{3^2}{2} \cdot \frac{1}{2^3} = \frac{3^2}{2^8} = \frac{9}{256};$

b) $\frac{8^2 \cdot 25^3 \cdot 21^5}{50^2 \cdot 14^5 \cdot 15^2} = \frac{\left(2^3\right)^2 \cdot \left(5^2\right)^3 \cdot (3\cdot 7)^5}{\left(2\cdot 5^2\right)^2 \cdot (2\cdot 7)^5 \cdot (3\cdot 5)^2} = \frac{2^6 \cdot 5^6 \cdot 3^5 \cdot 7^5}{2^2 \cdot 5^4 \cdot 2^5 \cdot 7^5 \cdot 3^2 \cdot 5^2} = \frac{3^3}{2} = \frac{27}{2};$

c) $\frac{(-6)^5 \cdot 15^4}{(-8)^2 \cdot 18^4 \cdot (-5)^5} = \frac{-(2\cdot 3)^5 \cdot (3\cdot 5)^4}{-\left(2^3\right)^2 \cdot \left(2\cdot 3^2\right)^4 \cdot 5^5} = \frac{2^5 \cdot 3^5 \cdot 3^4 \cdot 5^4}{2^6 \cdot 2^4 \cdot 3^8 \cdot 5^5} = \frac{3}{2^5 \cdot 5} = \frac{3}{160};$

d) $\frac{32 \cdot (-3)^8 \cdot 6^{-3}}{(-12)^5 \cdot 18^{-2}} = \frac{2^5 \cdot 3^8 \cdot \left(2\cdot 3^2\right)^2}{-\left(2^2\cdot 3\right)^5 \cdot (2\cdot 3)^3} = -\frac{2^5 \cdot 3^8 \cdot 2^2 \cdot 3^4}{2^{10} \cdot 3^5 \cdot 2^3 \cdot 3^3} = -\frac{3^4}{2^6} = -\frac{81}{64};$

e) $\frac{(-5)^3 \cdot 20^{-4}}{(-8)^{-3} \cdot 15^2 \cdot (-9)^{-2}} = \frac{5^3 \cdot \left(2^3\right)^3 \cdot \left(3^2\right)^2}{(3\cdot 5)^2 \cdot \left(2^2\cdot 5\right)^4} = \frac{5^3 \cdot 2^9 \cdot 3^4}{3^2 \cdot 5^2 \cdot 2^8 \cdot 5^4} = \frac{2\cdot 3^2}{5^3} = \frac{18}{125};$

f) $\frac{10^{12} - 10^9}{10^6} = \frac{10^9\left(10^3 - 1\right)}{10^6} = 10^3 \cdot (1000 - 1) = 999000;$

g) $\frac{3^7 - 3^6 - 3^5}{3^6 - 3^5} = \frac{3^5\left(3^2 - 3 - 1\right)}{3^5(3-1)} = \frac{5}{2};$

h) $\frac{3^7 + 3^4}{6^5 - 6^3} = \frac{3^4\left(3^3 + 1\right)}{6^3\left(6^2 - 1\right)} = \frac{3^4 \cdot 2^2 \cdot 7}{2^3 \cdot 3^3 \cdot 5 \cdot 7} = \frac{3}{2\cdot 5} = \frac{3}{10};$

2. Calcular:

a) $\frac{0{,}00000047 \cdot 10^4}{0{,}0001 \cdot 470 \cdot 10^9} = \frac{47 \cdot 10^{-8} \cdot 10^4}{10^{-4} \cdot 47 \cdot 10 \cdot 10^9} = \frac{1}{10^{10}};$

b) $\frac{64{,}5 \cdot 10^5 \cdot 43{,}21}{4.321 \cdot 0{,}645 \cdot 10^7} = \frac{645 \cdot 10^{-1} \cdot 10^5 \cdot 4.321 \cdot 10^{-2}}{4.321 \cdot 645 \cdot 10^{-3} \cdot 10^7} = \frac{1}{10^2};$

c) $\frac{7{,}2 \cdot 10^{-2} - 36 \cdot 10^{-3}}{0{,}0018} = \frac{72 \cdot 10^{-3} - 36 \cdot 10^{-3}}{18 \cdot 10^{-4}} = \frac{36 \cdot 10^{-3}(2-1)}{18 \cdot 10^{-4}} = 2 \cdot 10 = 20;$

3. Desarrollar:

a) $(2x-x^3)^4=(2x)^4-4(2x)^3x^3+6(2x)^2(x^3)^2-8x(x^3)^3+(x^3)^4=x^{12}-8x^{10}+24x^8-32x^6+16x^4$

b) $\left(2x^2+\frac{x}{2}\right)^4=\left(\frac{4x^2+x}{2}\right)^4=\frac{(4x^2+x)^4}{2^4}=\frac{(4x^2)^4+4(4x^2)^3x+6(4x^2)^2x^2+4\cdot 4x^2x^3+x^4}{16}=$

$=\frac{256x^8+256x^7+96x^6+16x^5+x^4}{16}=16x^8+16x^7+6x^6+x^5+\frac{x^4}{16};$

c) $\left(x^2-\frac{x}{3}\right)^5=\frac{(3x^2-x)^5}{3^5}=\frac{(3x^2)^5-5(3x^2)^4x+10(3x^2)^3x^2-10\cdot(3x^2)^2x^3+5\cdot 3x^2x^4-x^5}{243}=$

$=\frac{243x^{10}-405x^9+270x^8-90x^7+15x^6-x^5}{243}=x^{10}-\frac{5x^9}{9}+\frac{10x^8}{9}-\frac{10x^7}{27}+\frac{5x^6}{81}-\frac{x^5}{243}.$

4. Simplificar:

a) $\sqrt{20}-\sqrt{45}+\sqrt{80}=\sqrt{2^2\cdot 5}-\sqrt{3^2\cdot 5}+\sqrt{2^4\cdot 5}=2\sqrt{5}-3\sqrt{5}+4\sqrt{5}=3\sqrt{5};$

b)
$\sqrt{75}+\sqrt{12}-\sqrt{27}+\sqrt{48}=\sqrt{3\cdot 5^2}+\sqrt{2^2\cdot 3}-\sqrt{3^3}+\sqrt{2^4\cdot 3}=5\sqrt{3}+2\sqrt{3}-3\sqrt{3}+2^2\sqrt{3}=8\sqrt{3};$

c) $\sqrt{18}-\sqrt{48}+\sqrt{32}-\sqrt{12}=\sqrt{2\cdot 3^2}-\sqrt{2^4\cdot 3}+\sqrt{2^5}-\sqrt{2^2\cdot 3}=3\sqrt{2}-2^2\sqrt{3}+2^2\sqrt{2}-2\sqrt{3}=$

$=7\sqrt{2}-6\sqrt{3};$

d) $3\cdot\sqrt[3]{16}-2\cdot\sqrt[3]{250}+5\cdot\sqrt[3]{54}-4\cdot\sqrt[3]{2}=3\cdot\sqrt[3]{2^4}-2\cdot\sqrt[3]{2\cdot 5^3}+5\cdot\sqrt[3]{2\cdot 3^3}-4\cdot\sqrt[3]{2}=$

$=6\cdot\sqrt[3]{2}-10\cdot\sqrt[3]{2}+15\cdot\sqrt[3]{2}-4\cdot\sqrt[3]{2}=7\cdot\sqrt[3]{2};$

e) $\sqrt[3]{16}+\sqrt[3]{135}+\sqrt[3]{54}-\sqrt[3]{625}=\sqrt[3]{2^4}+\sqrt[3]{3^3\cdot 5}+\sqrt[3]{2\cdot 3^3}-\sqrt[3]{5^4}=2\cdot\sqrt[3]{2}+3\cdot\sqrt[3]{5}+3\cdot\sqrt[3]{2}-5\cdot\sqrt[3]{5}=$

$=5\cdot\sqrt[3]{2}-2\cdot\sqrt[3]{5}.$

5. Racionalizar y simplificar:

a) $\sqrt[3]{25}\cdot\sqrt{5}\cdot\sqrt[6]{\frac{1}{25}}=\frac{\sqrt[3]{5^2}\cdot\sqrt{5}}{\sqrt[3]{5}}=\sqrt[3]{5}\cdot\sqrt{5}=\sqrt[6]{5^2\cdot 5^3}=\sqrt[6]{5^5}=\sqrt[6]{3.125};$

b) $\frac{\sqrt[4]{8}\cdot 2^{-1}}{2\cdot\sqrt{2}}=\frac{\sqrt[4]{2^3}\cdot\sqrt{2}}{2\cdot 2\cdot\sqrt{2}\cdot\sqrt{2}}=\frac{\sqrt[4]{2^3\cdot 2^2}}{2^2\cdot 2}=\frac{\sqrt[4]{2^5}}{2^3}=\frac{2\cdot\sqrt[4]{2}}{8}=\frac{\sqrt[4]{2}}{4};$

c) $\dfrac{2\sqrt{3}-\sqrt{2}}{\sqrt{18}} = \dfrac{2\sqrt{3}-\sqrt{2}}{\sqrt{2\cdot 3^2}} = \dfrac{\left(2\sqrt{3}-\sqrt{2}\right)\cdot\sqrt{2}}{3\cdot\sqrt{2}\cdot\sqrt{2}} = \dfrac{2\sqrt{6}-2}{3\cdot 2} = \dfrac{2\cdot\left(\sqrt{6}-1\right)}{6} = \dfrac{\sqrt{6}-1}{3};$

d) $\dfrac{\sqrt{3}\cdot\sqrt[3]{9}}{\sqrt[4]{27}} = \dfrac{\sqrt{3}\cdot\sqrt[3]{3^2}\cdot\sqrt[4]{3}}{\sqrt[4]{3^3}\cdot\sqrt[4]{3}} = \dfrac{\sqrt[12]{3^6\cdot 3^8\cdot 3^3}}{\sqrt[4]{3^4}} = \dfrac{\sqrt[12]{3^{17}}}{3} = \dfrac{3\cdot\sqrt[12]{3^5}}{3} = \sqrt[12]{243};$

e) $\dfrac{\sqrt{\sqrt[3]{32}}\cdot\sqrt[4]{8}}{4\cdot\sqrt[3]{2}} = \dfrac{\sqrt[6]{2^5}\cdot\sqrt[4]{2^3}\cdot\sqrt[3]{2^2}}{4\cdot\sqrt[3]{2}\cdot\sqrt[3]{2^2}} = \dfrac{\sqrt[12]{2^{10}\cdot 2^9\cdot 2^8}}{4\cdot\sqrt[3]{2^3}} = \dfrac{\sqrt[12]{2^{27}}}{4\cdot 2} = \dfrac{\sqrt[4]{2^9}}{8} = \dfrac{2^2\cdot\sqrt[4]{2}}{8} = \dfrac{\sqrt[4]{2}}{2};$

f) $\dfrac{\sqrt[3]{9}\cdot\sqrt{3}}{\sqrt[3]{81}} = \dfrac{\sqrt[3]{3^2}\cdot\sqrt{3}}{\sqrt[3]{3^4}} = \dfrac{\sqrt[3]{3^2}\cdot\sqrt{3}\cdot\sqrt[3]{3^2}}{3\cdot\sqrt[3]{3}\cdot\sqrt[3]{3^2}} = \dfrac{\sqrt[6]{3^4\cdot 3^3\cdot 3^4}}{3\cdot\sqrt[3]{3^3}} = \dfrac{\sqrt[6]{3^{11}}}{3\cdot 3} = \dfrac{3\cdot\sqrt[6]{3^5}}{9} = \dfrac{\sqrt[6]{243}}{3};$

g) $\dfrac{12}{3-\sqrt{3}} = \dfrac{12\left(3+\sqrt{3}\right)}{\left(3-\sqrt{3}\right)\left(3+\sqrt{3}\right)} = \dfrac{12\left(3+\sqrt{3}\right)}{9-3} = 2\left(3+\sqrt{3}\right) = 6+2\sqrt{3};$

h) $\dfrac{14}{3\sqrt{2}-5} = \dfrac{14\left(3\sqrt{2}+5\right)}{\left(3\sqrt{2}-5\right)\left(3\sqrt{2}+5\right)} = \dfrac{14\left(3\sqrt{2}+5\right)}{18-25} = -2\left(3\sqrt{2}+5\right) = -6\sqrt{2}-10;$

i) $\dfrac{\sqrt{5}-1}{\sqrt{5}+1} = \dfrac{\left(\sqrt{5}-1\right)^2}{\left(\sqrt{5}+1\right)\left(\sqrt{5}-1\right)} = \dfrac{5-2\sqrt{5}+1}{5-1} = \dfrac{6-2\sqrt{5}}{4} = \dfrac{2\left(3-\sqrt{5}\right)}{4} = \dfrac{3-\sqrt{5}}{2};$

j) $\dfrac{13}{3\sqrt{2}+\sqrt{5}} = \dfrac{13\left(3\sqrt{2}-\sqrt{5}\right)}{\left(3\sqrt{2}+\sqrt{5}\right)\left(3\sqrt{2}-\sqrt{5}\right)} = \dfrac{13\left(3\sqrt{2}-\sqrt{5}\right)}{18-5} = 3\sqrt{2}-\sqrt{5};$

k) $\dfrac{12}{2\sqrt{3}+\sqrt{15}} = \dfrac{12\left(2\sqrt{3}-\sqrt{15}\right)}{\left(2\sqrt{3}+\sqrt{15}\right)\left(2\sqrt{3}-\sqrt{15}\right)} = \dfrac{12\left(2\sqrt{3}-\sqrt{15}\right)}{12-15} = -4\left(2\sqrt{3}-\sqrt{15}\right) = 4\sqrt{15}-8\sqrt{3}.$