

Tema 4. Polinomios y fracciones algebraicas

Ejercicios resueltos

1. Dados los polinomios: $P(x)=x^3+5x-3$, $Q(x)=2x^5+x^4+5x^2-1$ y $R(x)=\frac{2}{3}x^2-\frac{x-3}{5}$, efectuar:

a) $P(x)-Q(x)+R(x)$;

b) $Q(x)\cdot P(x)$

SOL:

$$\begin{aligned} \text{a) } P(x)-Q(x)+R(x) &= x^3+5x-3-2x^5-x^4-5x^2+1+\frac{2}{3}x^2-\frac{1}{5}x+\frac{3}{5} \\ &= -2x^5-x^4+x^3-\frac{13}{3}x^2+\frac{24}{5}x-\frac{7}{5} \end{aligned}$$

$$\begin{aligned} \text{b) } Q(x)\cdot P(x) &= (2x^5+x^4+5x^2-1)\cdot(x^3+5x-3) \\ &= 2x^8+10x^6-6x^5+x^7+5x^5-3x^4+5x^5+25x^3-15x^2-x^3-5x+3 \\ &= 2x^8+x^7+10x^6+4x^5-3x^4+24x^3-15x^2-5x+3 \end{aligned}$$

2. Calcular el cociente $C(x)$ y el resto $R(x)$ de las siguientes divisiones:

a) $(5x^3-7x^2+4):(x-3)$; b) $(2x^3+3x-1):(x+2)$

$$\text{SOL: a) } \begin{array}{r|rrrr} & 5 & -7 & 0 & 4 \\ 3 & & 15 & 24 & 72 \\ \hline & 5 & 8 & 24 & 76 \end{array}$$

$$C(x)=5x^2+8x+24 \text{ y } R(x)=76$$

$$\text{b) } \begin{array}{r|rrrr} & 2 & 0 & 3 & -1 \\ -2 & & -4 & 8 & -22 \\ \hline & 2 & -4 & 11 & -23 \end{array}$$

$$C(x)=2x^2-4x+11 \text{ y } R(x)=-23$$

3. Descomponer en factores los siguientes polinomios:

a) $3x-2$; b) $2x^2+3x-2$; c) x^2-4 ; d) $2x^2+4x$; e) x^2+4x+4 ; f) $4x^3-12x^2+9x$; g) $2x^3-5x^2-4x+3$;
h) $16x^4-65x^2+4$

$$\text{SOL: a) } 3x-2 = 3x - \frac{2 \cdot 3}{3} = 3 \left(x - \frac{2}{3} \right)$$

$$b) 2x^2 + 3x - 2 = 0; \quad x = \frac{-3 \pm \sqrt{9+16}}{4} = \frac{-3 \pm 5}{4} = \begin{cases} \frac{2}{4} = \frac{1}{2} \\ \frac{-8}{4} = -2 \end{cases} \Rightarrow 2x^2 + 3x - 2 = 2(x+2)\left(x - \frac{1}{2}\right)$$

$$c) x^2 - 4 = x^2 - 2^2 = (x+2)(x-2)$$

$$d) 2x^2 + 4x = 2x(x+2)$$

$$e) x^2 + 4x + 4 = x^2 + 2 \cdot x \cdot 2 + 2^2 = (x+2)^2$$

$$f) 4x^3 - 12x^2 + 9x = x(4x^2 - 12x + 9) = x[(2x)^2 - 2 \cdot 2x \cdot 3 + 3^2] = x(2x-3)^2 = 4x\left(x - \frac{3}{2}\right)^2$$

$$g) \begin{array}{c|cccc} & 2 & -5 & -4 & 3 \\ -1 & & -2 & 7 & -3 \\ \hline & 2 & -7 & 3 & 0 \end{array} \quad 2x^2 - 7x + 3 = 0; \quad x = \frac{7 \pm \sqrt{49-24}}{4} = \frac{7 \pm 5}{4} = \begin{cases} \frac{12}{4} = 3 \\ \frac{2}{4} = \frac{1}{2} \end{cases}$$

$$2x^3 - 5x^2 - 4x + 3 = (x+1)(2x^2 - 7x + 3) = 2(x+1)\left(x - \frac{1}{2}\right)(x-3)$$

$$h) 16(x^2)^2 - 65x^2 + 4 = 0; \quad x^2 = \frac{65 \pm \sqrt{4.225 - 256}}{32} = \frac{65 \pm 63}{32} = \begin{cases} x^2 = \frac{128}{32} = 4 \Rightarrow x = \pm\sqrt{4} = \pm 2 \\ x^2 = \frac{2}{32} = \frac{1}{16} \Rightarrow x = \pm\sqrt{\frac{1}{16}} = \pm \frac{1}{4} \end{cases}$$

$$16x^4 - 65x^2 + 4 = 16(x+2)\left(x + \frac{1}{4}\right)\left(x - \frac{1}{4}\right)(x-2)$$

4. Simplificar:

$$a) \frac{x^3 + x^2 - 8x - 12}{2x^3 - 2x^2 - 12x}; \quad b) \frac{x^3 - 4x}{3x^4 + x^3 - 12x^2 - 4x}; \quad c) \frac{x^4 - 4x^2}{3x^4 - x^3 - 12x^2 + 4x};$$

$$d) \frac{3x^4 + x^3 - 12x^2 - 4x}{16x^4 - 65x^2 + 4}; \quad e) \frac{9x^4 - 82x^2 + 9}{2x^4 + x^3 - 18x^2 - 9x}$$

$$SOL: a) \frac{x^3 + x^2 - 8x - 12}{2x^3 - 2x^2 - 12x} = \frac{(x+2)(x^2 - x - 6)}{2x(x^2 - x - 6)} = \frac{x+2}{2x}, \text{ ya que: } \begin{array}{c|cccc} & 1 & 1 & -8 & -12 \\ -2 & & -2 & 2 & 12 \\ \hline & 1 & -1 & -6 & 0 \end{array}$$

$$b) \frac{x^3 - 4x}{3x^4 + x^3 - 12x^2 - 4x} = \frac{x(x^2 - 4)}{x(3x^3 + x^2 - 12x - 4)} = \frac{(x+2)(x-2)}{3(x-2)\left(x + \frac{1}{3}\right)(x+2)} = \frac{1}{3x+1}, \text{ ya que:}$$

$$2 \left| \begin{array}{cccc} 3 & 1 & -12 & -4 \\ & 6 & 14 & 4 \\ \hline 3 & 7 & 2 & 0 \end{array} \right. \quad 3x^2 + 7x + 2 = 0; \quad x = \frac{-7 \pm \sqrt{49 - 24}}{6} = \frac{-7 \pm 5}{6} = \begin{cases} \frac{-2}{6} = -\frac{1}{3} \\ \frac{-12}{6} = -2 \end{cases}$$

c) $\frac{x^4 - 4x^2}{3x^4 - x^3 - 12x^2 + 4x} = \frac{x^2(x^2 - 4)}{x(3x^3 - x^2 - 12x + 4)} = \frac{x(x+2)(x-2)}{3(x-2)\left(x - \frac{1}{3}\right)(x+2)} = \frac{x}{3x-1}$, ya que:

$$2 \left| \begin{array}{cccc} 3 & -1 & -12 & 4 \\ & 6 & 10 & -4 \\ \hline 3 & 5 & -2 & 0 \end{array} \right. \quad 3x^2 + 5x - 2 = 0; \quad x = \frac{-5 \pm \sqrt{25 + 24}}{6} = \frac{-5 \pm 7}{6} = \begin{cases} \frac{2}{6} = \frac{1}{3} \\ \frac{-12}{6} = -2 \end{cases}$$

d) $\frac{3x^4 + x^3 - 12x^2 - 4x}{16x^4 - 65x^2 + 4} = \frac{3x(x-2)\left(x + \frac{1}{3}\right)(x+2)}{16(x+2)\left(x + \frac{1}{4}\right)\left(x - \frac{1}{4}\right)(x-2)} = \frac{3x^2 + x}{16x^2 - 1}$ (ver ejercicios 3.h) y 4.b))

e) $\frac{9x^4 - 82x^2 + 9}{2x^4 + x^3 - 18x^2 - 9x} = \frac{9x^4 - 82x^2 + 9}{x(2x^3 + x^2 - 18x - 9)} = \frac{9(x+3)\left(x + \frac{1}{3}\right)\left(x - \frac{1}{3}\right)(x-3)}{2x\left(x + \frac{1}{2}\right)(x+3)(x-3)} = \frac{9x^2 - 1}{2x^2 + x}$, por:

$$9(x^2)^2 - 82x^2 + 9 = 0; \quad x^2 = \frac{82 \pm \sqrt{6.724 - 324}}{18} = \frac{82 \pm 80}{18} = \begin{cases} x^2 = \frac{162}{18} = 9 \Rightarrow x = \pm\sqrt{9} = \pm 3 \\ x^2 = \frac{2}{18} = \frac{1}{9} \Rightarrow x = \pm\sqrt{\frac{1}{9}} = \pm\frac{1}{3} \end{cases}$$

$$3 \left| \begin{array}{cccc} 2 & 1 & -18 & -9 \\ & 6 & 21 & 9 \\ \hline 2 & 7 & 3 & 0 \end{array} \right. \quad 2x^2 + 7x + 3 = 0; \quad x = \frac{-7 \pm \sqrt{49 - 24}}{4} = \frac{-7 \pm 5}{4} = \begin{cases} \frac{-2}{4} = -\frac{1}{2} \\ \frac{-12}{4} = -3 \end{cases}$$

5. Calcular:

a) $\frac{2x+6}{x^2-3x} - \frac{x+5}{x^2-4x+3}$; b) $\frac{2}{x-1} - \frac{x+7}{x^2-1} - \frac{3}{x+1}$; c) $\frac{3}{x-3} - \frac{5}{x+3} - 2 \cdot \frac{x+6}{x^2-9}$

SOL: a) $x^2 - 4x + 3 = 0; \quad x = \frac{4 \pm \sqrt{16 - 12}}{2} = \frac{4 \pm 2}{2} = \begin{cases} \frac{6}{2} = 3 \\ \frac{2}{2} = 1 \end{cases} \Rightarrow \begin{cases} x^2 - 4x + 3 = (x-1)(x-3) \\ x^2 - 3x = x(x-3) \end{cases}$

$$\frac{2x+6}{x^2-3x} - \frac{x+5}{x^2-4x+3} = \frac{(x-1)(2x+6) - x(x+5)}{x(x-1)(x-3)} = \frac{2x^2+6x-2x-6-x^2-5x}{x(x-1)(x-3)} = \frac{x^2-x-6}{x(x-1)(x-3)}$$

$$= \frac{(x+2)(x-3)}{x(x-1)(x-3)} = \frac{x+2}{x^2-x}, \text{ ya que: } x^2-x-6=0; \quad x = \frac{1 \pm \sqrt{1+24}}{2} = \frac{1 \pm 5}{2} = \begin{cases} \frac{6}{2} = 3 \\ \frac{-4}{2} = -2 \end{cases}$$

$$\text{b) } \frac{2}{x-1} - \frac{x+7}{x^2-1} - \frac{3}{x+1} = \frac{2(x+1) - 1(x+7) - 3(x-1)}{(x+1)(x-1)} = \frac{2x+2-x-7-3x+3}{(x+1)(x-1)} = \frac{-2x-2}{(x+1)(x-1)}$$

$$= \frac{-2(x+1)}{(x+1)(x-1)} = \frac{2}{1-x}$$

$$\text{c) } \frac{3}{x-3} - \frac{5}{x+3} - 2 \cdot \frac{x+6}{x^2-9} = \frac{3(x+3) - 5(x-3) - 1 \cdot 2(x+6)}{(x+3)(x-3)} = \frac{3x+9-5x+15-2x-12}{(x+3)(x-3)}$$

$$= \frac{-4x+12}{(x+3)(x-3)} = \frac{-4(x-3)}{(x+3)(x-3)} = \frac{-4}{x+3}$$

6. Calcular:

$$\text{a) } \frac{3x-3}{x+1} \cdot \frac{x^2+2x+1}{x^2-1}; \quad \text{b) } \frac{x^3-6x^2+9x}{x^2-4} \cdot \frac{2x^2+3x-2}{x^2-3x};$$

$$\text{c) } \frac{3x+6}{x^2+4x+4} : \frac{x^2-2x}{x^2-4}; \quad \text{d) } \frac{x^2+2x+1}{x^3+2x^2} : \frac{3x+3}{x} \cdot \frac{3x^2+6x}{x^2-1}$$

$$\text{SOL: a) } \frac{3x-3}{x+1} \cdot \frac{x^2+2x+1}{x^2-1} = \frac{3(x-1)(x+1)^2}{(x+1)(x+1)(x-1)} = 3$$

$$\text{b) } \frac{x^3-6x^2+9x}{x^2-4} \cdot \frac{2x^2+3x-2}{x^2-3x} = \frac{x(x-3)^2 2(x+2) \left(x - \frac{1}{2}\right)}{(x+2)(x-2)x(x-3)} = \frac{(x-3)(2x-1)}{x-2} = \frac{2x^2-7x+3}{x-2}$$

$$\text{c) } \frac{3x+6}{x^2+4x+4} : \frac{x^2-2x}{x^2-4} = \frac{3(x+2)(x+2)(x-2)}{x(x-2)(x+2)^2} = \frac{3}{x}$$

$$\text{d) } \frac{x^2+2x+1}{x^3+2x^2} : \frac{3x+3}{x} \cdot \frac{3x^2+6x}{x^2-1} = \frac{(x+1)^2 x}{3(x+1)x^2(x+2)} \cdot \frac{3x(x+2)}{(x+1)(x-1)} = \frac{1}{x-1}$$